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Natural convection in a differentially heated enclosure filled with a micropolar fluid

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Abstract

A two-dimensional numerical simulation is conducted to analyze the steady laminar natural convective flow and heat transfer of micropolar fluids in a square enclosure. The vertical walls are kept at isothermal conditions, while horizontal walls are assumed to be insulated. Employing the finite difference method, computations are carried out to investigate the material parameter of the micropolar fluid, the Rayleigh and Prandtl numbers, both for weak and strong concentration cases. It was shown that micropolar fluids give lower heat transfer values than those of the Newtonian fluids. An increase of the material parameter, K is shown to decrease the heat transfer. The results for $K = 0$, which corresponds to the Newtonian fluid case are compared with those available in the existing literature, and an excellent agreement was obtained.

Keywords: Micropolar fluid; Natural convection; Enclosure; Material parameter; Rayleigh number; Prandtl number

1. Introduction

During recent years, the theory of micropolar fluids has received much attention and this is because the traditional Newtonian fluids cannot precisely describe the characteristics of the fluid flow with suspended particles. Studies on micropolar fluids have recently received considerable attention due to their application in a number of processes that occur in industry. Such applications include the extrusion of polymer fluids and real fluids with suspensions, solidification of liquid crystals, cooling of a metallic plate in a bath, animal bloods, porous media, turbulent shear flows, flow in capillaries and microchannels, and colloidal and suspension solutions, for example, for which the classical Navier–Stokes theory is inadequate. The concept of such fluids is to provide a mathematical model for the behavior of fluids taking into account the initial characteristics of the substructure particles which are allowed to undergo rotation. In recent years there exist several new developments in fluid mechanics that are concerned with structures within fluid, fluids which the classical theory has proved to be inadequate to describe their behavior. The simplest theory considered for structured fluids, the theory for micropolar fluids has been introduced by Eringen [1,2]. This theory contains six viscosity coefficients for compressible fluids and five for incompressible ones and there are two kinematics vector fields: the usual velocity field and an axial vector that represents the spin or the micro-rotation of the micropolar fluid particles which are assumed to be rigid, see Faltas and Saad [3]. Owing to its relatively mathematical simplicity, the micropolar fluids model has been also widely used in lubrication to investigate the polymer solutions in which the Newtonian lubricant is blended with small amount of long-chained additives.

The essence of the theory of micropolar fluid flow lies in the extension of the constitutive equations for Newtonian fluids so that more complex fluids can be described by this theory. In this theory, rigid particles contained in a small fluid volume element are limited to rotation about the center of the volume element described by the micro-rotation vector. This local rotation of the particles is in addition to the usual rigid body motion of the entire volume element. In the micropolar fluid theory, the laws of classical continuum mechanics are augmented with additional

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Nomenclature

g	acceleration due to gravity	$m\,s^{-2}$
j	microinertia density	m^2
K	non-dimensional material parameter	
L	length of the square cavity	m
n	constant	
$\overline{N}$	angular velocity	s^{-1}
Nu	local Nusselt number	
Nu_{av}	average Nusselt number	
Pr	Prandtl number	
Ra	Rayleigh number	
$\bar{t}$	time	s
T	fluid temperature	K
T_c	temperature of the cold wall	K
T_h	temperature of the hot wall	K
T_0	characteristic temperature	K
$\bar{u}, \bar{v}$	velocity components along $\bar{x}$ and $\bar{y}$ axes, respectively	$m\,s^{-1}$
$\bar{x}, \bar{y}$	Cartesian coordinates in the horizontal and vertical directions, respectively	m

Greek letters

α	thermal diffusivity	$m^2\,s^{-1}$
β	thermal expansion coefficient	K^{-1}
γ	spin gradient viscosity	$kg\,m\,s^{-1}$
κ	vortex viscosity	$kg\,m^{-1}\,s^{-1}$
θ	dimensionless temperature	
μ	dynamic viscosity	$kg\,m^{-1}\,s^{-1}$
ν	kinematic viscosity	$m^2\,s^{-1}$
ρ	fluid density	$kg\,m^{-3}$
$\overline{\omega}$	vorticity function	s^{-1}
ψ	non-dimensional stream function	

Subscripts

mid	midplane of the cavity
max	maximum value
min	minimum value

equations that account for the conservation of micro-inertia moments and the balance of first stress moments that arise due to consideration of the microstructure in a material, and also additional local constitutive parameters are introduced. Physically micropolar fluids may be described as the non-Newtonian fluids consisting of dumb-bell molecules or short rigid cylindrical elements, polymer fluids, fluid suspensions, animal blood, etc. The presence of dust or smoke, particularly in a gas, may also be modeled using micropolar fluid dynamics. The key points to note in the development of Eringen's microcontinuum mechanics are the introduction of new kinematic variables, e.g. the gyration tensor and microinertia moment tensor, and the addition of the concept of body moments, stress moments, and microstress averages to the classical continuum mechanics. However, a serious difficulty is encountered when this theory is applied to real, non-trivial flow problems; even for the linear theory, a problem dealing with simple microfluids must be formulated in terms of a system of nineteen partial differential equations in nineteen unknowns and the underlying mathematical problem is not easily amenable to solution. These special features for micropolar fluids were discussed in a comprehensive review paper of the subject and application of micropolar fluid mechanics by Ariman et al. [4]. Early studies along these lines may be found in the review article by Peddieson and McNitt [5], and in the recent books by Łukaszewicz [6] and Eringen [7]. However, to our best knowledge, the general constitutive theories governing the behavior of micropolar fluids have not been described experimentally yet. An experimental work is required to provide a means for determining the value of the parameters that describe such fluids. How does one experimentally determine the microrotation? How does one go about prescribing boundary conditions for microrotation? What is the experimental evidence for the specification of such a condition? Regarding specific liquid crystals there are many models which describe the constitutive theories of these liquids, such as those due to Leslie and Ericksen, De Gennes, and others (see Ariman et al. [4]). Even for such models there are difficulties with regard to prescribing boundary conditions. For instance, in the case of director theories one has the onerous task of specifying boundary conditions for directors. This is a kind of continuum mechanics, and many classical flows are being re-examined to determine the effects of fluid microstructure (Willson [8], Bergholz [9], etc.). However, Hoyt and Fabula [10] have shown experimentally that the fluids containing minute polymeric additives indicate considerable reduction of the skin friction (about 25–30%), a concept which can be well explained by the theory of micropolar fluids. Power [11] has shown that the fluid flowing in brain (CSF) is adequately modeled by micropolar fluids.

In many engineering applications and naturally occurring processes, natural convection plays an important role as a dominating mechanism. Besides its importance in such processes, due to the coupling of fluid flow and energy transport, the phenomenon of natural convection remains an interesting field of investigation. This fact is reflected by numerous studies in the existing literature dedicated to this topic during the past few decades. Some excellent comprehensive review articles on this subject are given by Ostrach [12], Yang [13], and Fusegi and Hyun [14].

Most of the previous studies on natural convection in enclosures have been related to Newtonian fluids. Despite the importance of the micropolar fluids mentioned above, there are only a few of research efforts on natural convection of these fluids in enclosures. Hsu and Chen [15] numerically investigated the Rayleigh–Benard convection of a micropolar fluid in an enclosure using the cubic spline collocation method. They performed parametric studies on the effects of microstructure of heat and fluid flow. It was found that the heat transfer rate of micropolar fluids was smaller than that of the Newtonian fluid. Natural convection of micropolar fluids in a completely partitioned enclosure heated from below was investigated by Hsu and Tsai [16]. In another work, Hsu et al. [17] studied natural convection of micropolar fluids in a tilting enclosure equipped with a single or multiple uniform heat sources. In a recent study, Aydin and Pop [18] numerically investigated the steady laminar natural convective flow and heat transfer of micropolar fluids in enclosures with a centrally located discrete heater in one of its sidewalls by applying a finite difference method.

As an extension of our previous study mentioned above [18], the aim of the present study is focused on analyzing the steady natural convective heat transfer of micropolar fluids in a square cavity with differentially heated vertical walls and adiabatic horizontal walls using an elegant theory of micropolar fluids. The effects of main governing parameters such as the Rayleigh number, Prandtl number and material parameter are studied and also compared with those for Newtonian fluids. A very good agreement has been found. To our best knowledge the present problem has not been studied previously.

2. Analysis

2.1. Mathematical formulation

Consider the natural convection flow in a square cavity of length L filled with a micropolar fluid, as shown in Fig. 1, where the coordinates $\bar{x}$ and $\bar{y}$ are chosen such that $\bar{x}$ measures the distance along the bottom horizontal wall, while $\bar{y}$ measures the distance along the left vertical wall, respectively. It is assumed that the horizontal walls are adiabatic, while the left and right vertical walls are kept at the constant temperatures T_h and T_c, respectively, where $T_h > T_c$. Under these assumptions, the basic unsteady equations of motion and energy are

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0 \tag{1}$$

$$\rho\left(\frac{\partial \bar{u}}{\partial \bar{t}} + \bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}\right) = -\frac{\partial \bar{p}}{\partial \bar{x}} + (\mu+\kappa)\left(\frac{\partial^2 \bar{u}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{u}}{\partial \bar{y}^2}\right) + \kappa\frac{\partial \overline{N}}{\partial \bar{y}} \tag{2}$$

$$\rho\left(\frac{\partial \bar{v}}{\partial \bar{t}} + \bar{u}\frac{\partial \bar{v}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}\right) = -\frac{\partial \bar{p}}{\partial \bar{y}} + (\mu+\kappa)\left(\frac{\partial^2 \bar{v}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{v}}{\partial \bar{y}^2}\right) - \kappa\frac{\partial \overline{N}}{\partial \bar{x}} + \rho g\beta(T - T_0) \tag{3}$$

$$\rho j\left(\frac{\partial \overline{N}}{\partial \bar{t}} + \bar{u}\frac{\partial \overline{N}}{\partial \bar{x}} + \bar{v}\frac{\partial \overline{N}}{\partial \bar{y}}\right) = \gamma\left(\frac{\partial^2 \overline{N}}{\partial^2 \bar{x}} + \frac{\partial^2 \overline{N}}{\partial^2 \bar{y}}\right) - 2\kappa\overline{N} + \kappa\left(\frac{\partial \bar{v}}{\partial \bar{x}} - \frac{\partial \bar{u}}{\partial \bar{y}}\right) \tag{4}$$

$$\frac{\partial T}{\partial \bar{t}} + \bar{u}\frac{\partial T}{\partial \bar{x}} + \bar{v}\frac{\partial T}{\partial \bar{y}} = \alpha\left(\frac{\partial^2 T}{\partial \bar{x}^2} + \frac{\partial^2 T}{\partial \bar{y}^2}\right) \tag{5}$$

subject to initial and boundary conditions

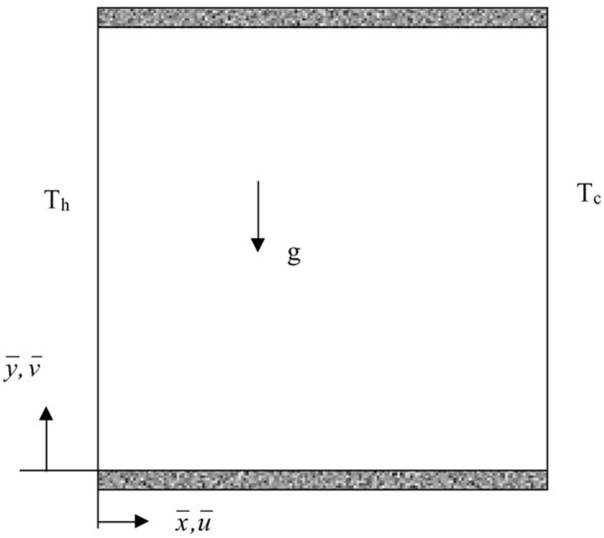

Fig. 1. The geometry of the problem.

$$\begin{aligned}
\bar{t} < 0:\quad & \bar{u} = \bar{v} = 0, \quad \overline{N} = 0 \\
& T = T_0 \quad \text{any} \quad 0 \leqslant \bar{x}, \bar{y} \leqslant L \\
\bar{t} \geqslant 0:\quad & \bar{u} = \bar{v} = 0, \quad \overline{N} = -n\frac{\partial \bar{u}}{\partial \bar{x}} \\
& T = T_h \quad \text{on} \quad \bar{x} = 0,\ 0 \leqslant \bar{y} \leqslant L \\
& \bar{u} = \bar{v} = 0, \quad \overline{N} = -n\frac{\partial \bar{u}}{\partial \bar{x}} \\
& T = T_c \quad \text{on} \quad \bar{x} = L,\ 0 \leqslant \bar{y} \leqslant L \\
& \bar{u} = \bar{v} = 0, \quad \overline{N} = -n\frac{\partial \bar{v}}{\partial \bar{y}} \\
& \frac{\partial T}{\partial \bar{y}} = 0 \quad \text{on} \quad \bar{y} = 0 \quad \text{and} \quad \bar{y} = L,\ 0 \leqslant \bar{x} \leqslant L
\end{aligned} \tag{6}$$

where $\bar{u}$ and $\bar{v}$ are the velocity components along $\bar{x}$ and $\bar{y}$ axes, T is the fluid temperature, $\overline{N}$ is the component of the microrotation vector normal to the $\bar{x} - \bar{y}$ plane, $\bar{t}$ is the time, g is the magnitude of the acceleration due to gravity, ρ is the density, μ is the dynamic viscosity, κ is the vortex viscosity, γ is the spin-gradient viscosity, j is the microinertia density, $T_0 = (T_h + T_c)/2$ is the characteristic temperature and n is a constant $0 \leqslant n \leqslant 1$. It should be mentioned that the case $n = 0$, called strong concentration of microelements (see Guram and Smith [19]), indicates $\overline{N} = 0$ near the walls. It represents concentrated particle flows in which the microelements close to the wall surface are unable to rotate. The case $n = 1/2$, on the other hand, indicates the vanishing of anti-symmetric part of the stress tensor and denotes weak concentration (see Jena and Mathur [20]). The case $n = 1$, as suggested by Peddieson [21] is used for the modeling of turbulent boundary layer flows. Further, we shall assume that γ has the following form as proposed by Ahmadi [22] and used by Rees and Pop [23] for the problem of free convection boundary layer flow over a vertical flat plate embedded in a micropolar fluid

$$\gamma = \left(\mu + \frac{\kappa}{2}\right)j = \mu\left(1 + \frac{K}{2}\right)j \tag{7}$$

where K is called the material parameter.

We eliminate now the pressure terms from Eqs. (2) and (3), and introduce the following non-dimensional variables

$$x = \bar{x}/L, \quad y = \bar{y}/L, \quad t = (\nu/L^2)\bar{t}$$
$$u = (L/\nu)\bar{u}, \quad v = (L/\nu)\bar{v}$$
$$\theta = (T - T_0)/(T_h - T_c)$$
$$N = (L^2/\nu)\overline{N}, \quad \omega = (L^2/\nu)\overline{\omega} \tag{8}$$

where ν is the kinematic viscosity and $\overline{\omega}$ is the vorticity function. Substituting (8) into Eqs. (1)–(5), we get

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} = -\omega \tag{9}$$

$$\frac{\partial\omega}{\partial t} + u\frac{\partial\omega}{\partial x} + v\frac{\partial\omega}{\partial y} = (1+K)\left(\frac{\partial^2\omega}{\partial x^2} + \frac{\partial^2\omega}{\partial y^2}\right) - K\left(\frac{\partial^2 N}{\partial y^2} + \frac{\partial^2 N}{\partial x^2}\right) + (Ra/Pr)\frac{\partial\theta}{\partial x} \tag{10}$$

$$\frac{\partial N}{\partial t} + u\frac{\partial N}{\partial x} + v\frac{\partial N}{\partial y} = \left(1+\frac{K}{2}\right)\left(\frac{\partial^2 N}{\partial x^2} + \frac{\partial^2 N}{\partial y^2}\right) - 2KN + K\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) \tag{11}$$

$$\frac{\partial\theta}{\partial t} + u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y} = \frac{1}{Pr}\left(\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2}\right) \tag{12}$$

where $j = L^2$ defines the length scale (see Rees and Bassom [24]), $Pr = \nu/\alpha$ is the Prandtl number, $Ra = g\beta(T_h - T_c)L^3/\alpha\nu$ is the Rayleigh number and ψ is the non-dimensional stream function which is defined in the usual way as

$$u = \frac{\partial\psi}{\partial y}, \quad v = -\frac{\partial\psi}{\partial x} \tag{13}$$

The initial and boundary conditions (6) become

$$\begin{aligned}
t < 0:\quad & u = v = 0, \quad N = 0 \\
& \theta = 0 \quad \text{any} \quad 0 \leqslant x, y \leqslant 1 \\
t \geqslant 0:\quad & u = v = 0, \quad N = -n\frac{\partial u}{\partial x} \\
& \theta = 0.5 \quad \text{on} \quad x = 0,\ 0 \leqslant y \leqslant 1 \\
& u = v = 0, \quad N = -n\frac{\partial u}{\partial x} \\
& \theta = -0.5 \quad \text{on} \quad x = 1,\ 0 \leqslant x \leqslant 1 \\
& u = v = 0, \quad N = -n\frac{\partial v}{\partial y} \\
& \frac{\partial\theta}{\partial y} = 0 \quad \text{on} \quad y = 0 \quad \text{and} \quad y = 1,\ 0 \leqslant x \leqslant 1
\end{aligned} \tag{14}$$

It is worth mentioning that for $K = 0$, Eqs. (9), (10) and (12) describe the classical problem of natural convection of a Newtonian fluid in a differentially heated square cavity, first considered by Vahl Davis [25].

The quantities of physical interest in this problem are also the local, Nu, and the average, Nu_{av}, Nusselt numbers at the vertical walls, which are given by

$$Nu = -\left(\frac{\partial\theta}{\partial x}\right)_{x=0,1}, \qquad Nu_{\text{av}} = -\int_0^1 Nu\,\mathrm{d}y \tag{15}$$

2.2. Method of solution

The numerical solutions to the systems of coupled partial differential equations (9)–(12) under the boundary conditions (14) are obtained using the finite-difference method. The vorticity transport, microrotation and energy equations are solved using the alternating direction implicit method and the stream function equation is solved by the successive overrelaxation (SOR) method. The overrelaxation parameter is chosen to be 1.8 for stream function solutions. In order to avoid divergence in the solution of the energy, microrotation and vorticity equations, an underrelaxation parameter of 0.5 is employed. Hybrid differencing is used with the convective terms, and central differences are used with diffusive and buoyancy terms. First-order-accurate forward differences are used with the time derivative. The following criterion is employed to check for the steady state solution:

$$\sum_{i,j}\left|\phi_{i,j}^{k+1} - \phi_{i,j}^{k}\right| \leqslant \varepsilon \tag{16}$$

where Φ stands for ψ, ω, N or θ; k refers to time; and i and j refer to space coordinates. The value of ε is chosen as 10^{-5}. Convergence of iterations for the stream function solution is obtained at each time step. The time step used in the computations is varied between 0.00001 and 0.004, depending on the Rayleigh number and mesh size. All the computations are carried out on a PC. After a grid refinement study, the majority of calculations presented here were made using a 31×31 non-uniform and non-staggered grid structure, which was constructed using finer grid spacing near the walls and coarser spacing in the interior of the cavity. The results obtained using a finer grid 61×61 do not reveal discernible changes in the predicted heat transfer and flow field. More details and validity of the numerical procedure can be found in Refs. [26,27].

3. Results and discussion

Computations are carried out for the following values of the governing parameter both for the weak concentration ($n = 0$) and for the strong concentration ($n = 0.5$) cases: $Pr = 0.01, 0.1, 0.71, 1, 10$; $Ra = 10^3, 10^4, 10^5$ and 10^6; $K = 0, 0.1, 0.5$ and 2. Interestingly, no difference is observed between the results of the weak and strong concentration cases. This is attributed to the symmetrical boundary conditions for the microrotation both in x and y directions. Therefore, we present here results only for the case of weak concentration case ($n = 0$). We should notice again that the case of $K = 0$ represents the Newtonian fluid. Initially, for this case, predicted results are compared with the benchmark solution of Vahl Davis [25] for the same geometry

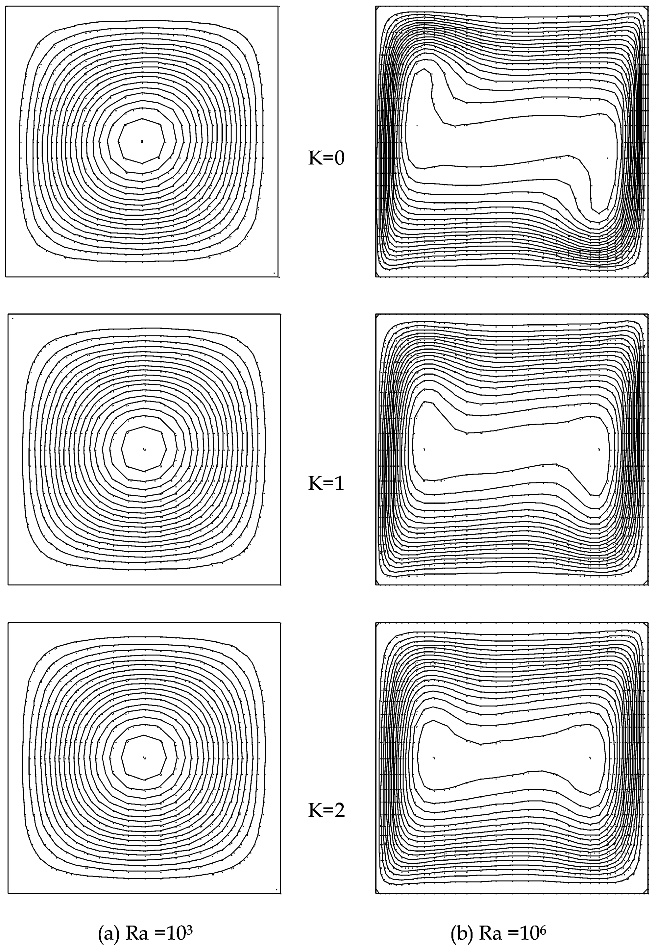

Fig. 2. Streamlines for different value of the material parameter, K and Rayleigh number, Ra when $Pr = 0.71$ and $n = 0$.

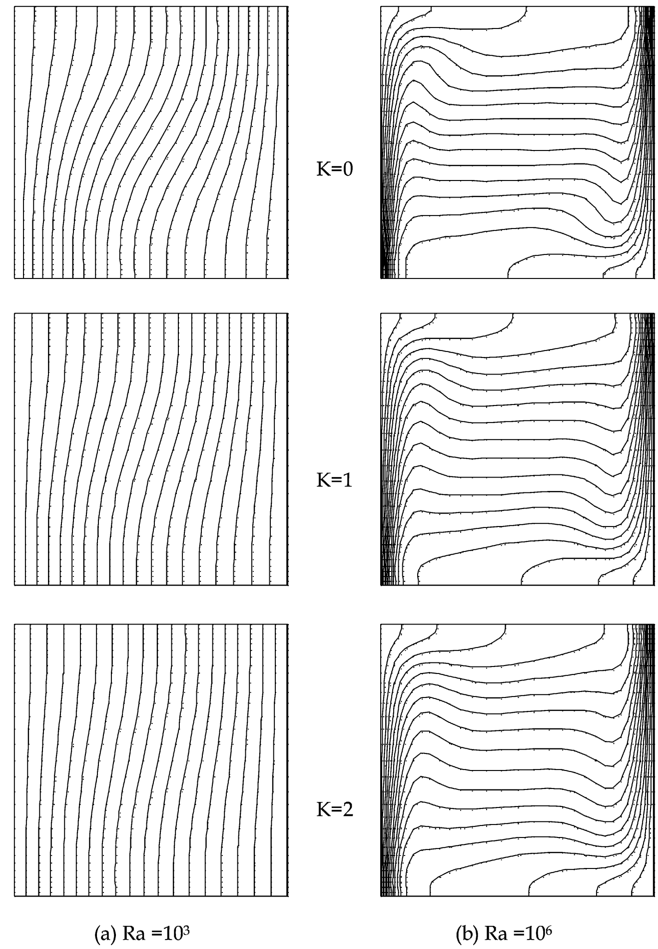

Fig. 3. Isotherms for different values of the material parameter, K and Rayleigh number, Ra when $Pr = 0.71$ and $n = 0$.

Table 1
Comparison of the results for $K = 0$ (Newtonian fluid) with those given by Ref. [24]

Ra	10^4			10^6		
	Present study	Ref. [24]	Difference (%)	Present study	Ref. [24]	Difference (%)
ψ_{mid}	5.087	5.071	0.310	16.445	16.320	0.760
ψ_{max}	5.087	5.071	0.310	16.954	16.750	1.200
U_{max}	16.225	16.178	0.290	65.874	64.630	1.890
V_{max}	19.645	19.617	0.140	215.350	219.360	1.860
Nu_{av}	2.234	2.243	0.400	8.945	8.800	1.620
Nu_{max}	3.531	3.528	0.080	18.254	17.925	1.800
Nu_{min}	0.589	0.586	0.510	0.975	0.989	1.440

examined here (i.e. a square cavity with differentially heated vertical walls and adiabatic horizontal walls). This is also a check for the validity of the computer code developed. A comparison between the results of the present study and the benchmark study for Rayleigh number values of $Ra = 10^4$ and 10^6 with $Pr = 0.71$ is shown in Table 1, where ψ_{mid}, ψ_{max}, U_{max}, V_{max}, N_{av}, Nu_{max} and Nu_{min} refer to the stream function at the midplane of the cavity, the maximum value of the stream function, the maximum horizontal velocity on the vertical midplane, the maximum vertical velocity on the horizontal midplane, the average (mean) Nusselt number at the hot wall, the maximum value of the local Nusselt number at the hot wall and the minimum value of the local Nusselt number at the hot wall, respectively. As can be seen, the deviations from the benchmark solutions are very small, which gives great confidence to the computer code used in this paper.

In the following, the effect of the material parameter K is studied. For each value of the Rayleigh number, the material parameter is increased and its effect on the momentum and energy transport is shown. Figs. 2, 3 and 4 show the streamlines, the isotherms, and the vorticity contours, respectively for the ranges of K considered with $Ra = 10^3$ and 10^6, $Pr = 0.71$. As expected, increased Rayleigh number numbers result in intensified circulation inside the enclosure and thinner thermal boundary layers near the heated and cooled walls, which lead to enhanced momentum and heat transfers, respectively. However, for a fixed value of the Rayleigh and Prandtl numbers, it is too difficult to see and explain the effect of the parameter K on heat and fluid flow from the figures above since related functions are very concentrated near the heated and cooled boundaries. Based on the examination of these figures, one may say that, at a fixed value of Ra and Pr, the effect of K seems to be negligible. This is misleading and therefore, heat transfer results are suggested to be analyzed. The effect of varying Ra on the aver-

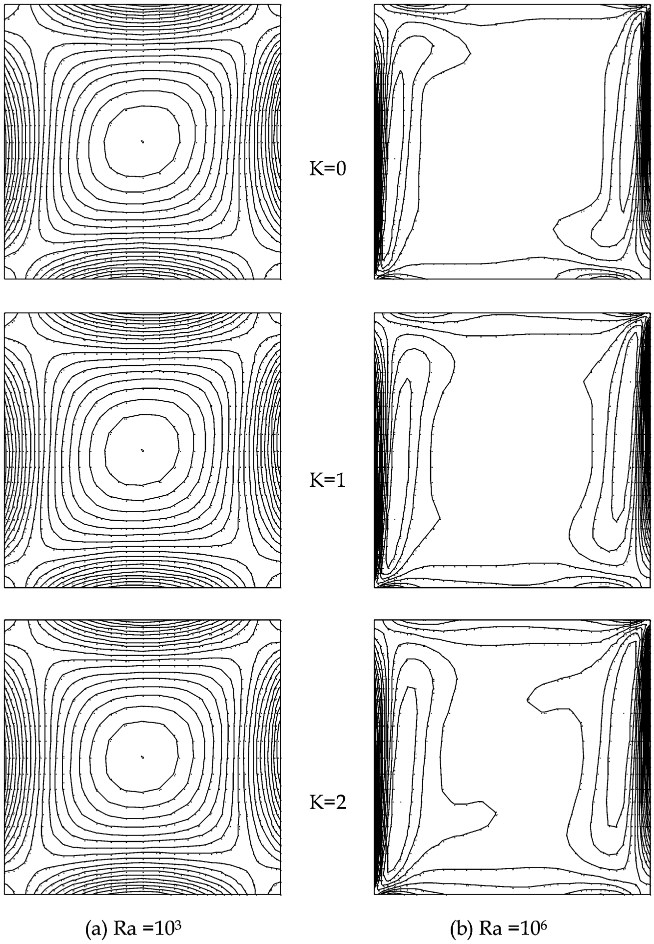

Fig. 4. Vorticity contours for different value of the material parameter, K and Rayleigh number, Ra when $Pr = 0.71$ and $n = 0$.

Table 2
The effect of K on the average Nusselt number Nu_{av} for different values of Ra when $Pr = 0.71$ and $n = 0$

Nu_{av}				
K	Ra			
	10^3	10^4	10^5	10^6
0	1.118	2.234	4.486	8.945
0.5	1.057	1.947	4.033	7.984
1	1.034	1.771	3.729	7.433
2	1.016	1.545	3.314	6.673

age Nusselt number, N_{av}, at the heated wall is shown in Fig. 5 for some values of K and a fixed value of $Pr = 0.71$. As seen for a fixed value of Ra, an increase in K reduces the heat transfer or average Nusselt number. In addition, the Newtonian fluid ($K = 0$) is found to have higher average heat transfer rates than a micropolar fluid ($K \neq 0$). This is because an increase in the vortex viscosity would result in an increase in the total viscosity of the fluid flow, thus decreasing the heat transfer. This is in agreement with the results reported by Kumari and Nath [28], and Chiu and Chou [29]. The values of N_{av} shown in Fig. 5 are also given in Table 2 for $Pr = 0.71$. Here it is also questioned how the decreasing effect of the material parameter, K on the heat transfer varies with the Prandtl number, Pr. Fig. 6 illus-

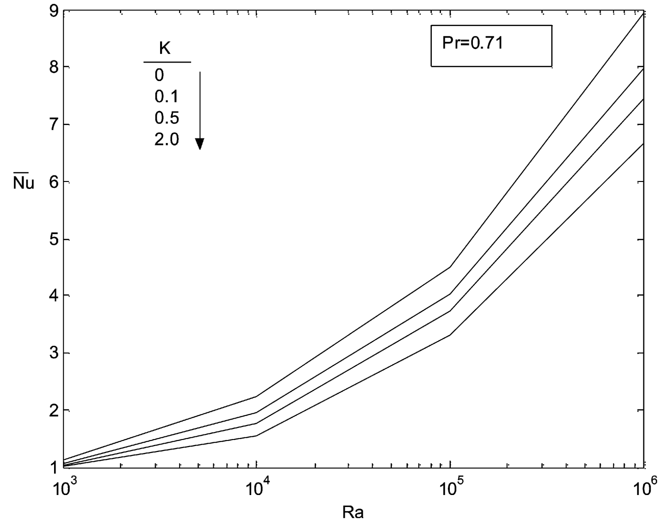

Fig. 5. The effect of Rayleigh number, Ra on the average Nusselt number, Nu_{av}, for $Pr = 0.71$ when $n = 0$.

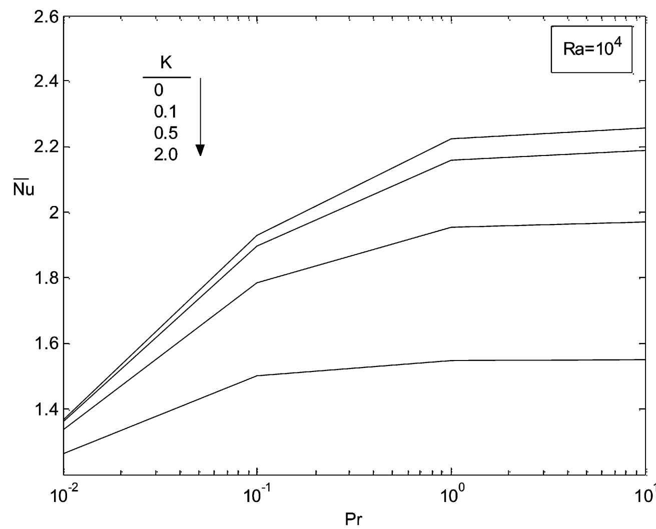

Fig. 6. The effect of Prandtl number, Pr on the average Nusselt number, Nu_{av}, for the Rayleigh number $Ra = 10^4$ when $n = 0$.

trates the effect of the Prandtl number on the heat transfer, for a fixed value of the Rayleigh number, $Ra = 10^4$, and for different values of the material parameter K. As known from the existing literature (see Bejan [30]), for low-Pr fluids ($Pr < 1$), the approximate expression for the averaged Nusselt number Nu_{av} is given in the form of $Nu_{av} \sim (Ra\,Pr)^{1/4}$, while it is given by $Nu_{av} \sim Ra^{1/4}$ for high-Pr fluids ($Pr > 1$). The results obtained here are, therefore, consistent with these above scale analysis results given by Bejan [30]. For each value of Pr, again, the effect of the material parameter K is found to decrease the heat transfer. It is also seen from Fig. 6 that for given values of Pr and Ra, the decrease of the heat transfer with the increase of K is more significant for higher values of Ra. We also notice that the rate of heat transfer increases with Pr. This is to be expected, because a larger Prandtl number results in a thinner thermal boundary layer with a corresponding large temperature gradient at the wall, and hence a large heat transfer.

4. Conclusions

In this study, the natural convection heat transfer of micropolar fluids in a differentially heated square enclosure is computationally studied using the finite difference method. Simulations are performed to investigate the effects of the Rayleigh number, Ra, Prandtl number, Pr and the material parameter, K on the momentum and heat transfer for a weak concentration particles of the micropolar fluid ($n = 0$). As expected, it is found that the average Nusselt number increases with increasing Rayleigh and Prandtl numbers. On the other hand, it is disclosed that an increase at the material parameter reduces the heat transfer.

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